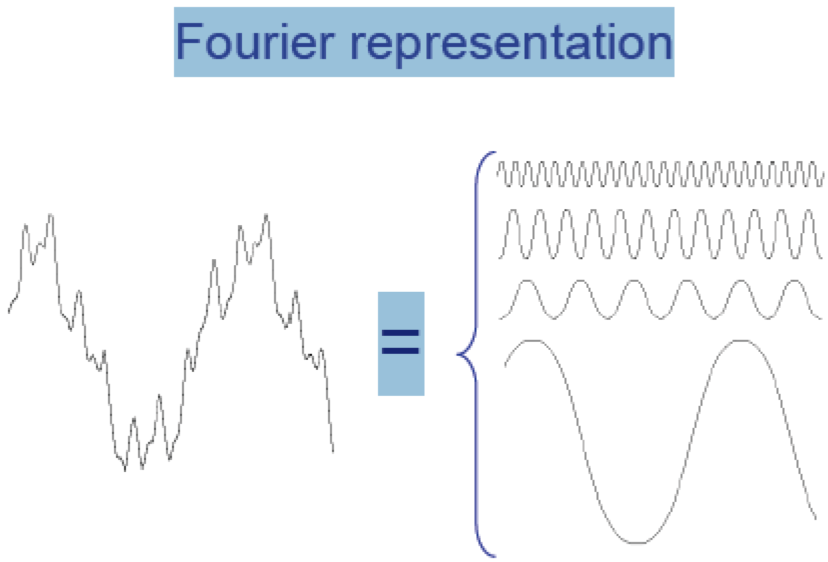
Image Enhancement in the Frequency Domain

Jean Baptiste Joseph Fourier (1768~1830), French mathematician



Fourier Series

Any periodically repeated function can be expressed of the sum of sines/cosines of different frequencies,each multiplied by a different coefficient.

Fourier Transform

Finite curves can be expressed as the integral of sines/cosines multiplied by a weighing function.It is wildly used in signal processing field

Fourier Series/Transform can be reconstructed completely via an inverse process.

Introduction to the Fourier Transform and the Frequency Domain

One dimensional Fourier transform

Discrete Fourier Transform

$$F(u)=\frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \text{ for } u = 0, 1, 2, ..., M-1.$$

Inverse Discrete Fourier Transform

$$f(x)=\sum_{x=0}^{M-1} F(u)e^{j2\pi ux/M} \text{ for } x = 0, 1, 2, ..., M-1.$$

Fourier spectra in 1D

– The absolute value of the complex function in the Fourier domain is the amplitude spectrum

$$F(u) = |F(u)|e^{-j\phi(u)}$$

$$|F(u)| = \left[R^2(u) + I^2(u)\right]^{1/2}$$

The angle defined by the inverse tangent of the ratio between the imaginary and the real components of the complex function is the phase spectrum.

$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right]$$

The square of the amplitude spectrum is the power spectrum

$$\begin{aligned} P(u) &= |F(u)|^2 \\ &= R^2(u) + I^2(u). \end{aligned}$$

2 D Discrete Fourier Transform (DFT)

The Discrete Time Fourier Transform of f(x,y) for x = 0, 1, 2…M-1 and y = 0,1,2…N-1, denoted by F(u, v), is given by the equation:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

for u = 0, 1, 2…M-1 and v = 0, 1, 2…N-1.

Inverse DFT

• It is really important to keep in mind that the Fourier transform is completely reversible

• The inverse DFT is given by:

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

for x = 0, 1, 2…M-1 and y = 0, 1, 2…N-1

Fourier spectra in 2D

We define the Fourier spectrum, phase angle, and power spectrum as in the previous section:

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

$$\begin{aligned} P(u,v) &= |F(u,v)|^2 \\ &= R^2(u,v) + I^2(u,v) \end{aligned}$$

where $R(u,v)$ and $I(u,v)$ are the real and imaginary parts of $F(u,v)$, respectively.

DFT properties:

The value of the DFT in the origin is the mean value of the function f(x,y).

$$F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y),$$

If f is real its DFT is conjugate symmetric.

$$F(u,v) = F^*(-u,-v)$$

Thus the Fourier spectrum is symmetric.

$$|F(u,v)| = |F(-u,-v)|.$$

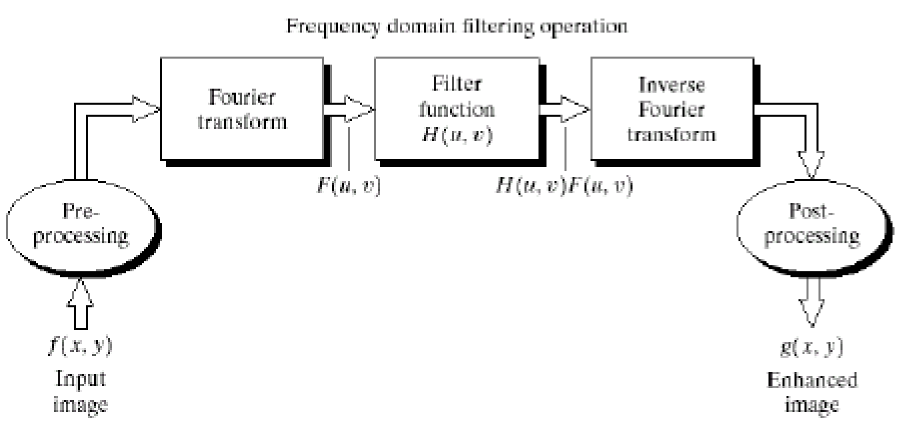
Shifts the origin of F(u, v) to (M/2, N/2)

$$\Im\left[f(x,y)(-1)^{x+y}\right] = F(u - M/2, v - N/2)$$

Basics of filtering in the frequency domain

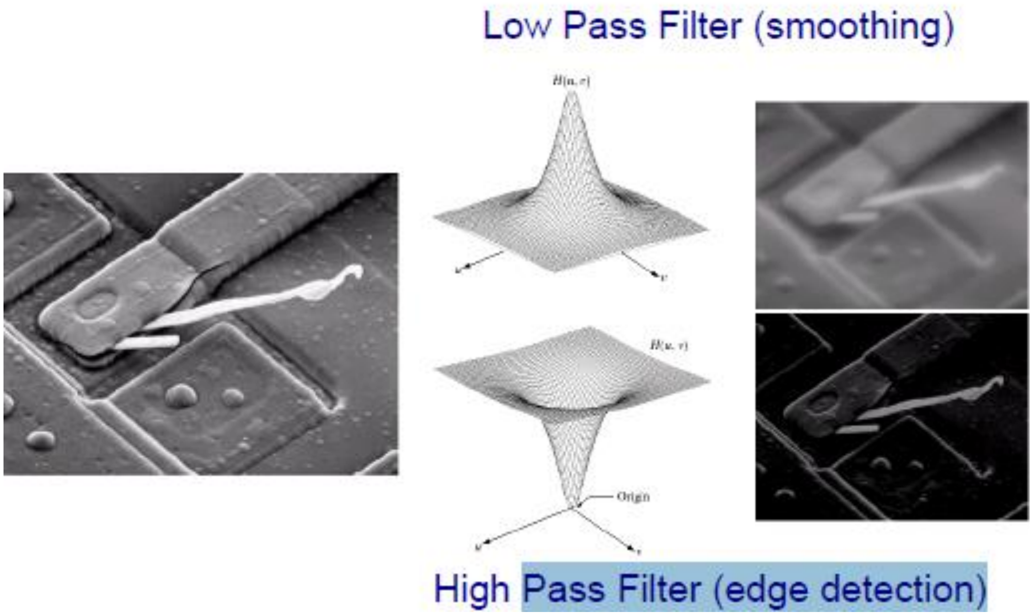
To filter an image in the frequency domain:

Compute F(u,v) the DFT of the image.
Multiply F(u,v) by a filter function H(u,v).
Compute the inverse DFT of the result.



Some Basic Frequency Domain Filters

Low Pass Filter (smoothing)
High Pass Filter (edge detection)



Filtering in Fourier domain

Multiply the input image by $(-1)^{x+y}$ to center the transform.

$$\Im\left[f(x,y)(-1)^{x+y}\right] = F(u - M/2, v - N/2)$$

Compute $F(u, v)$, the DFT of the image

Multiply $F(u, v)$ by a *filter* function $H(u, v)$.

Compute the inverse DFT of the result

Obtain the real part of the result

Multiply the result by $(-1)^{x+y}$.

$$G(u, v) = H(u, v)F(u, v).$$

H(u,v) is the filter transfer function, which is the DFT of the filter impulse response.
The implementation consists in multiplying point-wise the filter H(u,v) with the function F(u,v).
Real filters are called zero phase shift filters because they don’t change the phase of F(u,v).

Filtered image

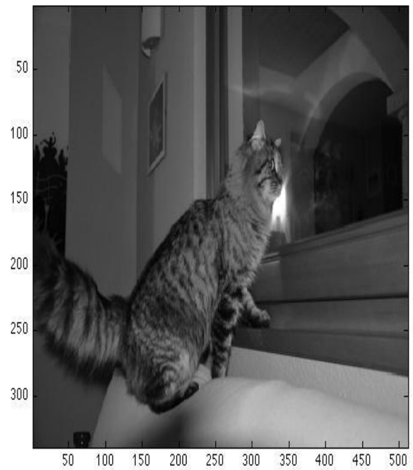
The filtered image is obtained by taking the inverse DFT of the resulting image.

$$\text{Filtered Image} = \Im^{-1}[G(u, v)].$$

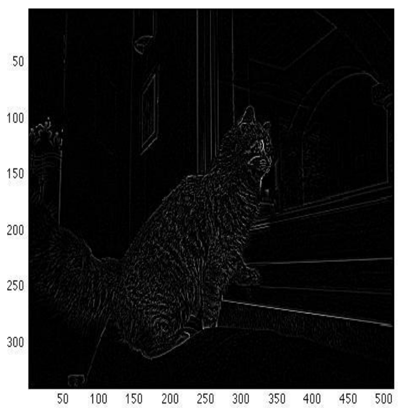
It can happen that the filtered image has spurious imaginary components even though the original image f(x,y) and the filter h(x,y) are real. These are due to numerical errors and are neglected.
The final result is thus the real part of the filtered image.

Smoothing: low pass filtering

Edge detection: high-pass filtering



Edge detection: greylevel image
Filtered image



Frequency Domain Filters

The basic model for filtering is:

$$G(u,v) = H(u,v)F(u,v)$$

where F(u,v) is the Fourier transform of the image being filtered and H(u,v) is the filter transform function.

Filtered image

$$f(x,y) = \Im^{-1}\{F(u,v)\}$$

Smoothing is achieved in the frequency domain by dropping out the high frequency components

–Low pass (LP) filters – only pass the low frequencies, drop the high ones.

– High-pass (HP) filters – olny pass the frequencies above a minimum value.

Reference:

Digital Image Processing 3rd ed. - R. Gonzalez, R. Woods

THANK YOU….