CMS-A-DSE-B--1-TH: Operation Research (O.R.)
Lesson XI: Network Scheduling

Introduction

Any project involves planning, scheduling and controlling a number of interrelated activities with use of limited resources, namely, men, machines, materials, money and time. The projects may be extremely large and complex such as construction of a power plant, a highway, a shopping complex, ships and aircraft, introduction of new products and research and development projects. It is required that managers must have a dynamic planning and scheduling system to produce the best possible results and also to react immediately to the changing conditions and make necessary changes in the plan and schedule. A convenient analytical and visual technique of PERT and CPM prove extremely valuable in assisting the managers in managing the projects.

Both the techniques use similar terminology and have the same purpose. PERT stands for Project Evaluation and Review Technique developed during 1950s. The technique was developed and used in conjunction with the planning and designing of the Polaris missile project. CPM stands for Critical Path Method which was developed by DuPont Company and applied first to the construction projects in the chemical industry. Though both PERT and CPM techniques have similarity in terms of concepts, the basic difference is, PERT is used for analysis of project scheduling problems. CPM has single time estimate and PERT has three time estimates for activities and uses probability theory to find the chance of reaching the scheduled time.

Project management generally consists of three phases:

1. ***Planning:*** Planning involves setting the objectives of the project. Identifying various activities to be performed and determining the requirement of resources such as men, materials, machines, etc. The cost and time for all the activities are estimated, and a network diagram is developed showing sequential interrelationships (predecessor and successor) between various activities during the planning stage.

2. ***Scheduling:*** Based on the time estimates, the start and finish times for each activity are worked out by applying forward and backward pass techniques, critical path is identified, along with the slack and float for the non-critical paths.

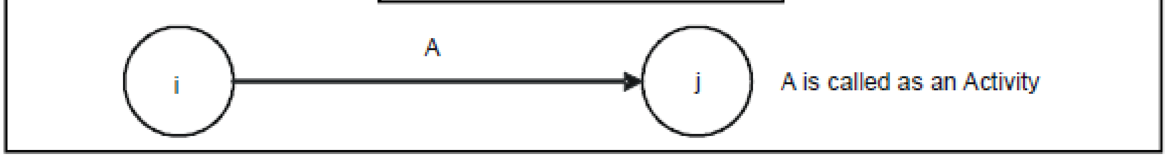
3. ***Controlling:*** Controlling refers to analyzing and evaluating the actual progress against the plan. Reallocation of resources, crashing and review of projects with periodical reports are carried out.

12.1 CPM/PERT Network Components

CPM/PERT networks contains two major components

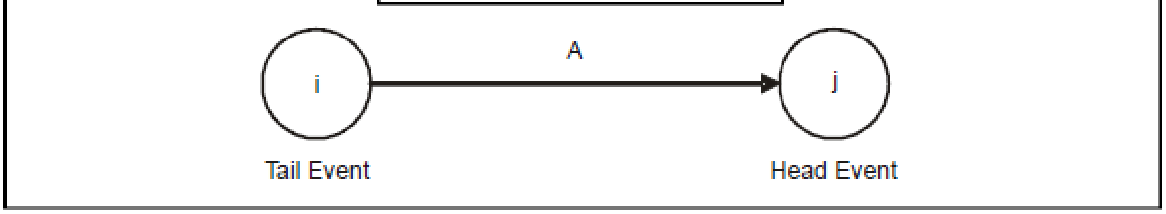
1. *Activity:* An activity represents an action and consumption of resources (time, money, energy) required to complete a portion of a project. Activity is represented by an arrow, (Figure 12.1):

Figure 12.1: An Activity



2. *Event:* An event (or node) will always occur at the beginning and end of an activity. The event has no resources and is represented by a circle. The ith event and jth event are the tail event and head event respectively, (Figure 12.2).

Figure 12.2: An Event



Merge and Burst Events

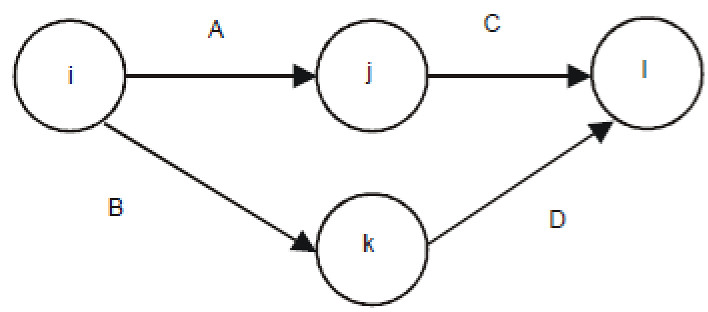
One or more activities can start and end simultaneously at an event (Figure 12.3 a, b).

Figure 12.3

Preceding and Succeeding Activities

Activities performed before given events are known as preceding activities (Figure 12.4), and activities performed after a given event are known as succeeding activities.

Figure 12.4: Preceding and Succeeding Activities

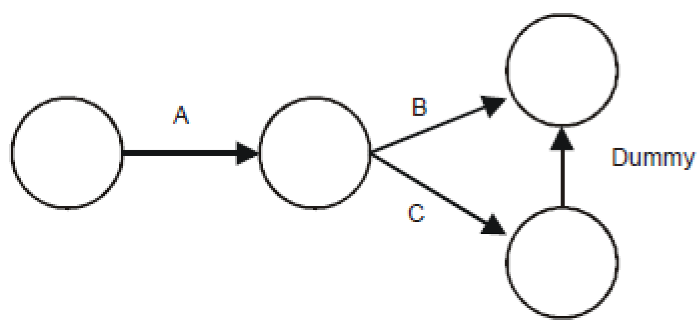


Activities A and B precede activities C and D respectively.

Dummy Activity

An imaginary activity which does not consume any resource and time is called a dummy activity. Dummy activities are simply used to represent a connection between events in order to maintain a logic in the network. It is represented by a dotted line in a network, see Figure 12.5.

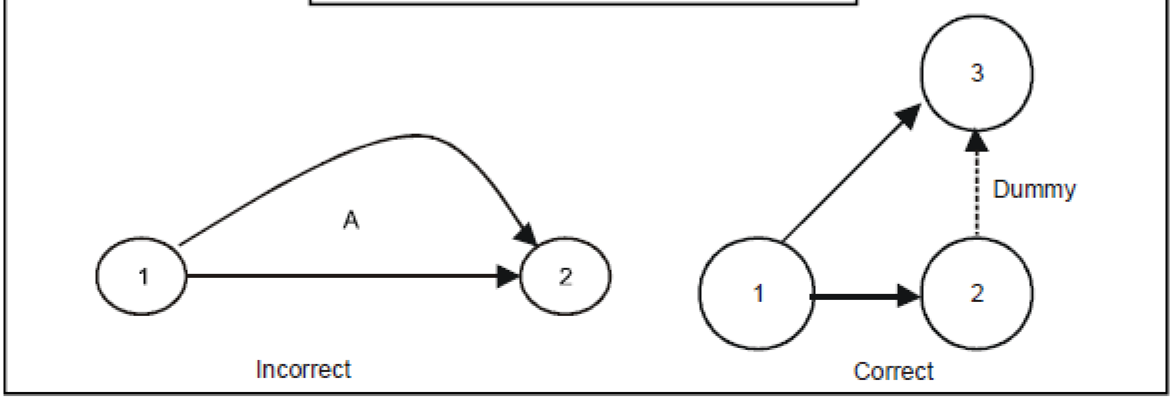
Figure 12.5: Dummy Activity



Errors to be avoided in Constructing a Network

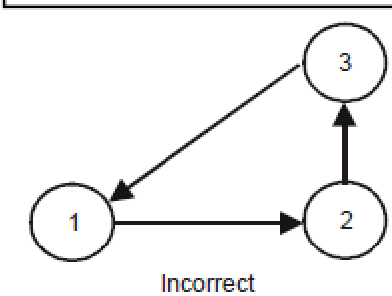
1. Two activities starting from a tail event must not have a same end event. To ensure this, it is absolutely necessary to introduce a dummy activity, as seen in Figure 12.6.

Figure 12.6: Correct and Incorrect Activities



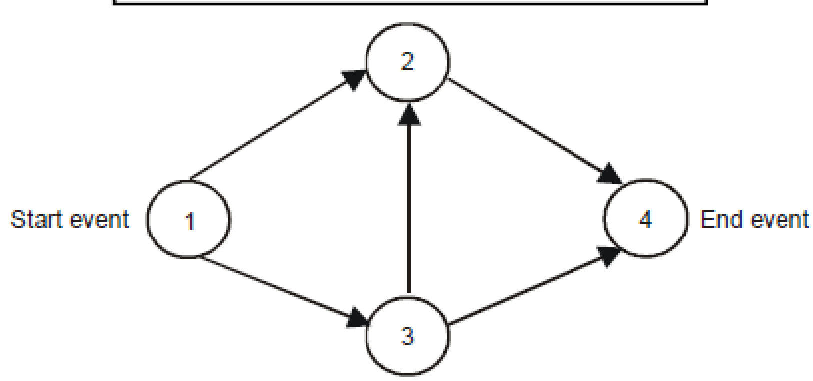
2. Looping error should not be formed in a network, as it represents performance of activities repeatedly in a cyclic manner, as shown below in Figure 12.7.

Figure 12.7: Looping Error



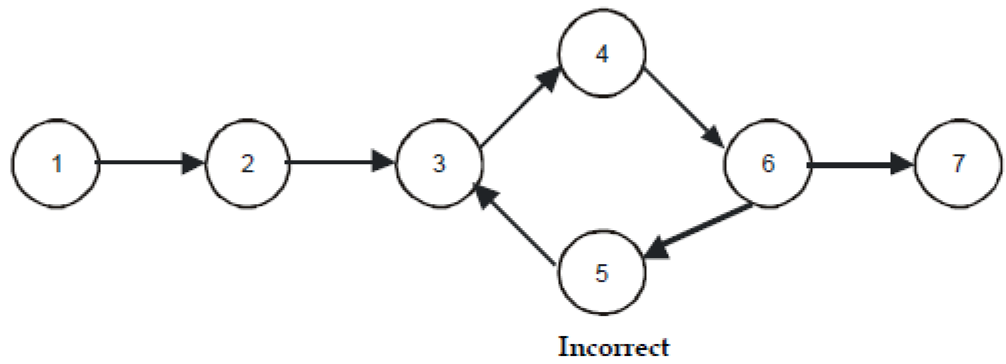
3. In a network, there should be only one start event and one ending event as shown below, in Figure 12.8.

Figure 12.8: Only One Start and End Event



4. The direction of arrows should flow from left to right avoiding mixing of direction as shown in Figure 12.9.

Figure 12.9: Wrong Direction of Arrows

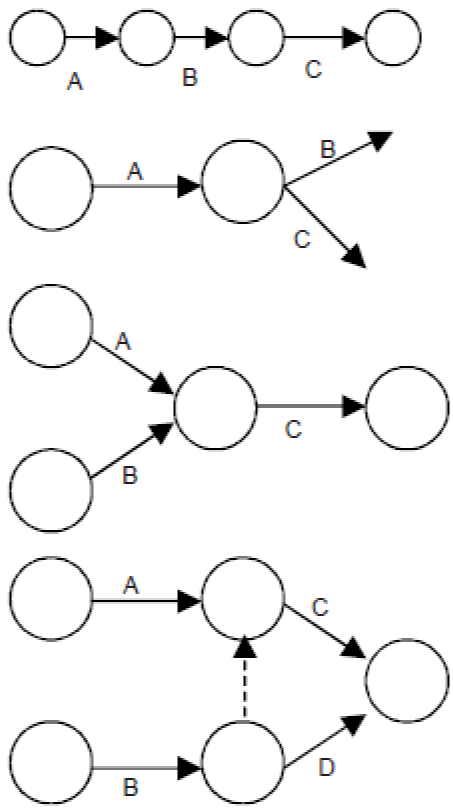


12.2 Rules in Constructing a Network

1. No single activity can be represented more than once in a network. The length of an arrow has no significance.
2. The event numbered 1 is the start event and an event with highest number is the end event. Before an activity can be undertaken, all activities preceding it must be completed. That is, the activities must follow a logical sequence (or interrelationship) between activities.
3. In assigning numbers to events, there should not be any duplication of event numbers in a network.
4. Dummy activities must be used only if it is necessary to reduce the complexity of a network.
5. A network should have only one start event and one end event.

Some conventions of network diagram is shown in Figure 12.10 (a), (b), (c), (d) below:

Figure 12.10 (a), (b), (c), (d): Some Conventions followed in Making Network Diagrams



(a) Activity B can be performed only after completing activity A, and activity C can be performed only after completing activity B.

(b) Activities B and C can start simultaneously only after completing A.

(c) Activities A and B must be completed before start of activity C.

(d) Activity C must start only after completing activities A and B. But activity D can start after completion of activity B.

Notes **Procedure for Numbering the Events using Fulkerson's Rule**

Step 1: Number the start or initial event as 1.

Step 2: From event 1, strike off all outgoing activities. This would have made one or more events as initial events (event which do not have incoming activities). Number that event as 2.

Step 3: Repeat step 2 for event 2, event 3 and till the end event. The end event must have the highest number.

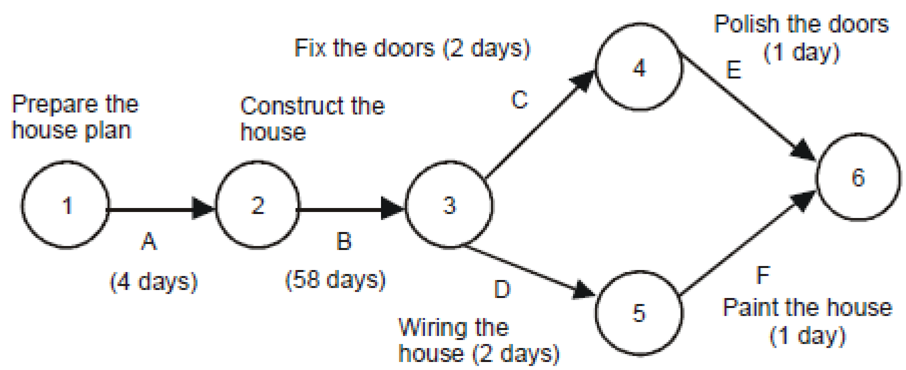
Example: Draw a network for a house construction project. The sequence of activities with their predecessors are given in Table 12.1, below.

Table 12.1: Sequence of Activities for House Construction Project

Name of the activity	Starting and finishing event	Description of activity	Predecessor	Time duration (days)
A	(1,2)	Prepare the house plan	--	4
B	(2,3)	Construct the house	A	58
C	(3,4)	Fix the door/windows	B	2
D	(3,5)	Wiring the house	B	2
E	(4,6)	Paint the house	C	1
F	(5,6)	Polish the doors/ windows	D	1

Solution:

Figure 12.11: Network Diagram Representing House Construction Project



The network diagram in Figure 12.11 shows the procedure relationship between the activities. Activity A (preparation of house plan), has a start event 1 as well as an ending event 2. Activity B (Construction of house) begins at event 2 and ends at event 3. The activity B cannot start until activity A has been completed. Activities C and D cannot begin until activity B has been completed, but they can be performed simultaneously. Similarly, activities E and F can start only after completion of activities C and D respectively. Both activities E and F finish at the end of event 6.

Example: Consider the project given in Table 12.2 and construct a network diagram.

Table 12.2: Sequence of Activities for Building Construction Project

Activity	Description	Predecessor
A	Purchase of Land	-
B	Preparation of building plan	-
C	Level or clean the land	A
D	Register and get approval	A, B
E	Construct the building	C
F	Paint the building	D

Solution:

The activities C and D have a common predecessor A. The network representation shown in Figure 12.12 (a), (b) violates the rule that no two activities can begin and end at the same events. It appears as if activity B is a predecessor of activity C, which is not the case. To construct the network in a logical order, it is necessary to introduce a dummy activity as shown in Figure 12.13.

Figure 12.12: Network Representing the Error

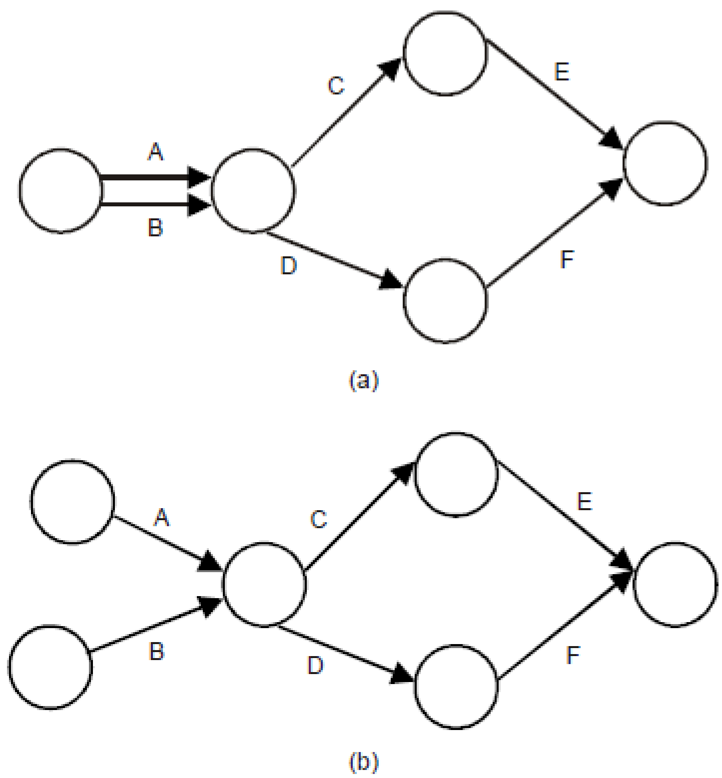
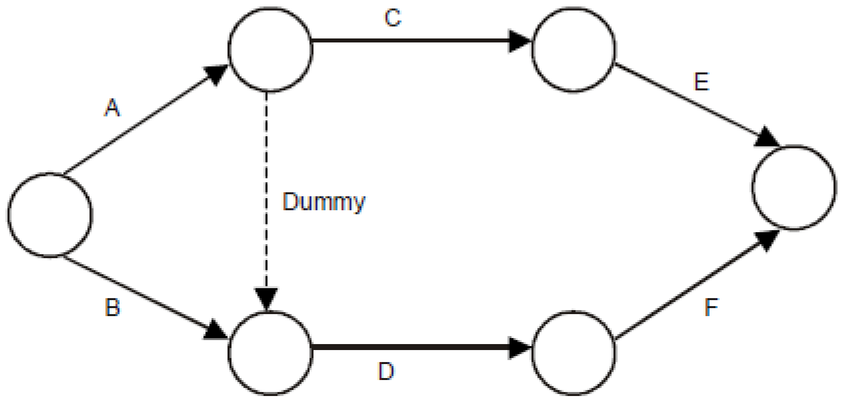


Figure 12.13: Correct Representation of Network using Dummy Activity



Example: Construct a network for a project whose activities and their predecessor relationship are given in Table 12.3.

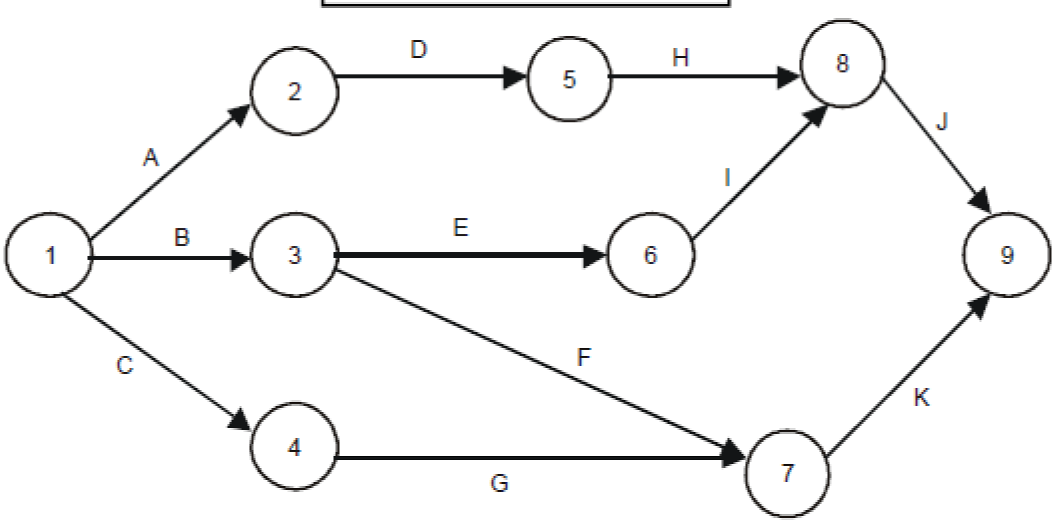
Table 12.3: Activity Sequence for a Project

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor	-	-	-	A	B	B	C	D	E	H, I	F, G

Solution:

The network diagram for the given problem is shown in Figure 12.14 with activities A, B and C starting simultaneously.

Figure 12.14: Network Diagram



Task Draw a network diagram for a project given in Table.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Immediate Predecessor	-	A	B	A	D	C, E	D	D	H	H	F, H	G, J

12.3 Critical Path Method (CPM)

The critical path for any network is the longest path through the entire network. Since all activities must be completed to complete the entire project, the length of the critical path is also the shortest time allowable for completion of the project. Thus if the project is to be completed in that shortest time, all activities on the critical path must be started as soon as possible. These activities are called critical activities. If the project has to be completed ahead of the schedule, then the time required for at least one of the critical activity must be reduced. Further, any delay in completing the critical activities will increase the project duration.

The activity, which does not lie on the critical path, is called non-critical activity. These non-critical activities may have some slack time. The slack is the amount of time by which the start of an activity may be delayed without affecting the overall completion time of the project. But a critical activity has no slack. To reduce the overall project time, it would require more resources (at extra cost) to reduce the time taken by the critical activities to complete.

12.3.1 Time Estimates: Earliest Time and Latest Time

Before the critical path in a network is determined, it is necessary to find the earliest and latest time of each event to know the earliest expected time (T_E) at which the activities originating from the event can be started and to know the latest allowable time (T_L) at which activities terminating at the event can be completed.

Forward Pass Computations (to calculate Earliest, Time T_E)

Procedure

Step 1: Begin from the start event and move towards the end event.

Step 2: Put $T_E = 0$ for the start event.

Step 3: Go to the next event (i.e. node 2) if there is an incoming activity for event 2, add calculate T_E of previous event (i.e. event 1) and activity time. Note: If there are more than one incoming activities, calculate T_E for all incoming activities and take the maximum value. This value is the T_E for event 2.

Step 4: Repeat the same procedure from step 3 till the end event.

Backward Pass Computations (to calculate Latest Time T_L)

Procedure

Step 1: Begin from end event and move towards the start event. Assume that the direction of arrows is reversed.

Step 2: Latest Time T_L for the last event is the earliest time. T_E of the last event.

Step 3: Go to the next event, if there is an incoming activity, subtract the value of T_L of previous event from the activity duration time. The arrived value is T_L for that event. If there are more than one incoming activities, take the minimum T_E value.

Step 4: Repeat the same procedure from step 2 till the start event.

Determination of Float and Slack Times

As discussed earlier, the non-critical activities have some slack or float. The *float* of an activity is the amount of time available by which it is possible to delay its completion time without extending the overall project completion time.

For an activity i = j, let

t_{ij} = duration of activity

T_E = earliest expected time

T_L = latest allowable time

ES_{ij} = earliest start time of the activity

EF_{ij} = earliest finish time of the activity

LS_{ij} = latest start time of the activity

LF_{ij} = latest finish time of the activity

Total float TF_{ij}: The total float of an activity is the difference between the latest start time and the earliest start time of that activity.

$$TF_{ij} = LS_{ij} - ES_{ij} \quad (1)$$

or

$$TF_{ij} = (T_L - T_E) - t_{ij} \quad (2)$$

Free Float FF_{ij}: The time by which the completion of an activity can be delayed from its earliest finish time without affecting the earliest start time of the succeeding activity is called free float.

$$FF_{ij} = (E_j - E_i) - t_{ij} \quad (3)$$

$$FF_{ij} = \text{Total float} - \text{Head event slack}$$

Independent Float IF_{ij}: The amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its latest finish time.

$$IF_{ij} = (E_j - L_i) - t_{ij} \quad (4)$$

$$IF_{ij} = \text{Free float} - \text{Tail event slack}$$

Where tail event slack = L_i-E_i

Caution The negative value of independent float is considered to be zero.

Critical Path: After determining the earliest and the latest scheduled times for various activities, the minimum time required to complete the project is calculated. In a network, among various paths, the longest path which determines the total time duration of the project is called the *critical path*. The following conditions must be satisfied in locating the critical path of a network.

An activity is said to be critical only if both the conditions are satisfied.

1. $T_L - T_E = 0$
2. $T_{Lj} - t_{ij} - T_{Ej} = 0$

Example: A project schedule has the following characteristics as shown in Table 12.4

Table 12.4: Project Schedule

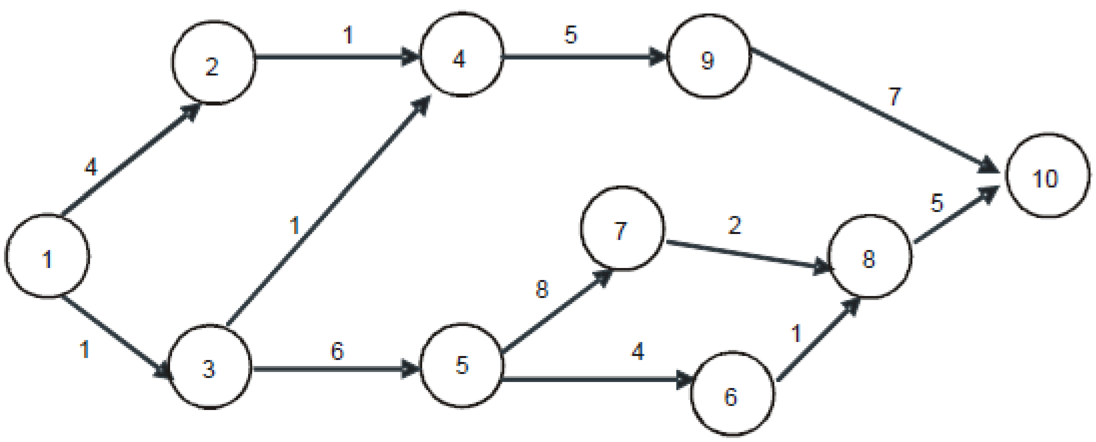
Activity	Name	Time	Activity	Name	Time (days)
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

1. Construct a network diagram.
2. Compute T_E and T_L for each activity.
3. Find the critical path.

Solution:

1. From the data given in the problem, the activity network is constructed as shown in Figure 5.6.

Figure 12.15: Activity Network Diagram



2. To determine the critical path, compute the earliest, time T_E and latest time T_L for each of the activity of the project. The calculations of T_E and T_L are as follows:

   To calculate T_E for all activities,

$T_{E1} = 0$

$T_{E2} = T_{E1} + t_{1,2} = 0 + 4 = 4$

$T_{E3} = T_{E1} + t_{1,3} = 0 + 1 = 1$

$T_{E4} = \max(T_{E2} + t_{2,4} \text{ and } T_{E3} + t_{3,4})$

$= \max(4 + 1 \text{ and } 1 + 1) = \max(5, 2)$

$= 5$ days

$T_{E5} = T_{E3} + t_{3,6} = 1 + 6 = 7$

$T_{E6} = T_{E5} + t_{5,6} = 7 + 4 = 11$

$T_{E7} = T_{E5} + t_{5,7} = 7 + 8 = 15$

$T_{E8} = \max(T_{E6} + t_{6,8} \text{ and } T_{E7} + t_{7,8})$

$= \max(11 + 1 \text{ and } 15 + 2) = \max(12, 17)$

$= 17$ days

$T_{E9} = T_{E4} + t_{4,9} = 5 + 5 = 10$

$T_{E10} = \max(T_{E9} + t_{9,10} \text{ and } T_{E8} + t_{8,10})$

$= \max(10 + 7 \text{ and } 17 + 5) = \max(17, 22)$

$= 22$ days

To calculate T_L for all activities

T_{L10} $= T_{E10} = 22$

T_{L9} $= T_{E10} - t_{9}, 10 = 22 - 7 = 15$

T_{L8} $= T_{E10} - t_{8,10} = 22 - 5 = 17$

T_{L7} $= T_{E8} - t_{7,8} = 17 - 2 = 15$

T_{L6} $= T_{E8} - t_{6,8} = 17 - 1 = 16$

T_{L5} $= \min (T_{E6} - t_{5,6}$ and $T_{E7} - t_{5,7})$

$= \min (16 - 4$ and $15 - 8) = \min (12, 7)$

$= 7$ days

T_{L4} $= T_{L9} - t_{4,9} = 15 - 5 = 10$

T_{L3} $= \min (T_{L4} - t_{3,4}$ and $T_{L5} - t_{3,5})$

$= \min (10 - 1$ and $7 - 6) = \min (9, 1)$

$= 1$ day

T_{L2} $= T_{L4} - t_{2,4} = 10 - 1 = 9$

T_{L1} $= \text{Min} (T_{L2} - t_{1,2}$ and $T_{L3} - t_{1,3})$

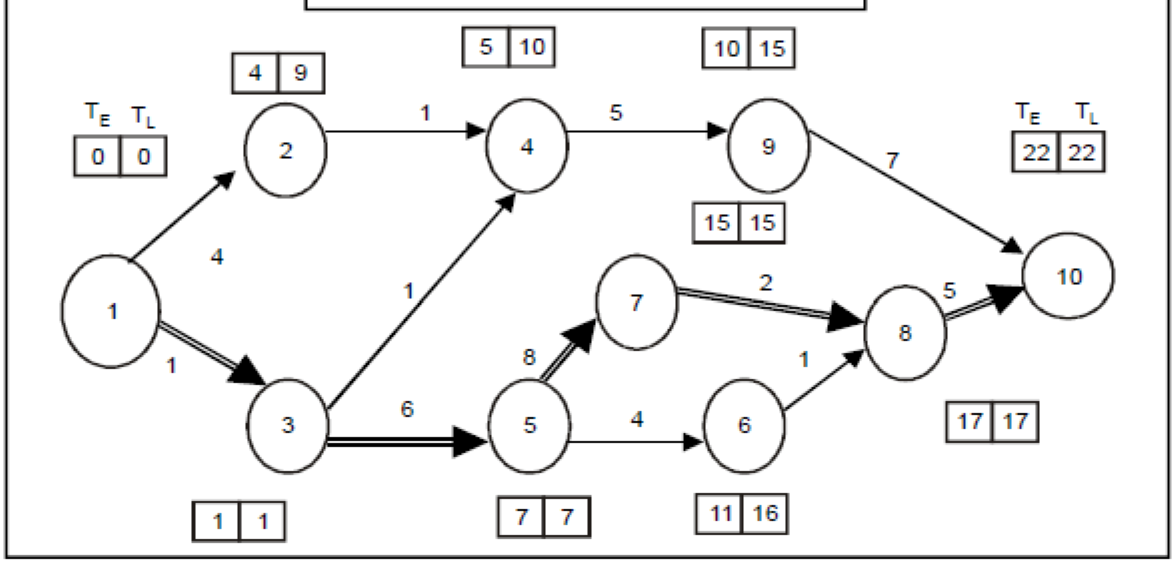
$= \text{Min} (9 - 4$ and $1 - 1) = 0$

Table 12.5: Various Activities and their Floats

Activity	Activity Name	Normal Time	Earliest Time		Latest Time		Total Float
			Start	Finish	Start	Finish	
1-2	A	4	0	4	5	9	5
1-3	B	1	0	1	0	1	0
2-4	C	1	4	5	9	10	5
3-4	D	1	1	2	9	10	8
3-5	E	6	1	7	1	7	0
4-9	F	5	5	10	10	15	5
5-6	G	4	7	11	12	16	5
5-7	H	8	7	15	7	15	0
6-8	I	1	11	12	16	17	5
7-8	J	2	15	17	15	17	0
8-10	K	5	17	22	19	22	0
9-10	L	7	10	17	15	22	5

3. From the table 12.5, we observe that the activities 1 – 3, 3 – 5, 5 – 7,7 – 8 and 8 – 10 are critical activities as their floats are zero.

Figure 12.16: Critical Path of the Project



The critical path is 1-3-5-7-8-10 (shown in double line in Figure 12.16) with the project duration of 22 days.

Example: The following Table 12.6 gives the activities in construction project and time duration.

Table 12.6: Project Schedule with Time Duration

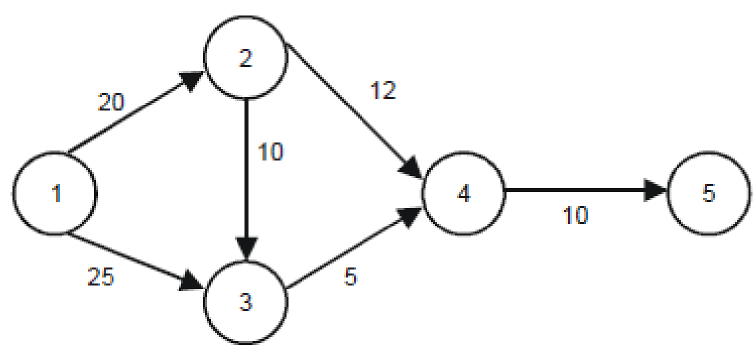
Activity	Preceding Activity	Normal time (days)
1-2	-	20
1-3	-	25
2-3	1-2	10
2-4	1-2	12
3-4	1-3,2-3	5
4-5	2-4,3-4	10

1. Draw the activity network of the project.
2. Find the total float and free float for each activity.

Solution:

1. From the activity relationship given, the activity network is shown in Figure 12.17 below:

Figure 12.17: Activity Network Diagram



2. The total and free floats for each activity are calculated as shown in Table 12.7.

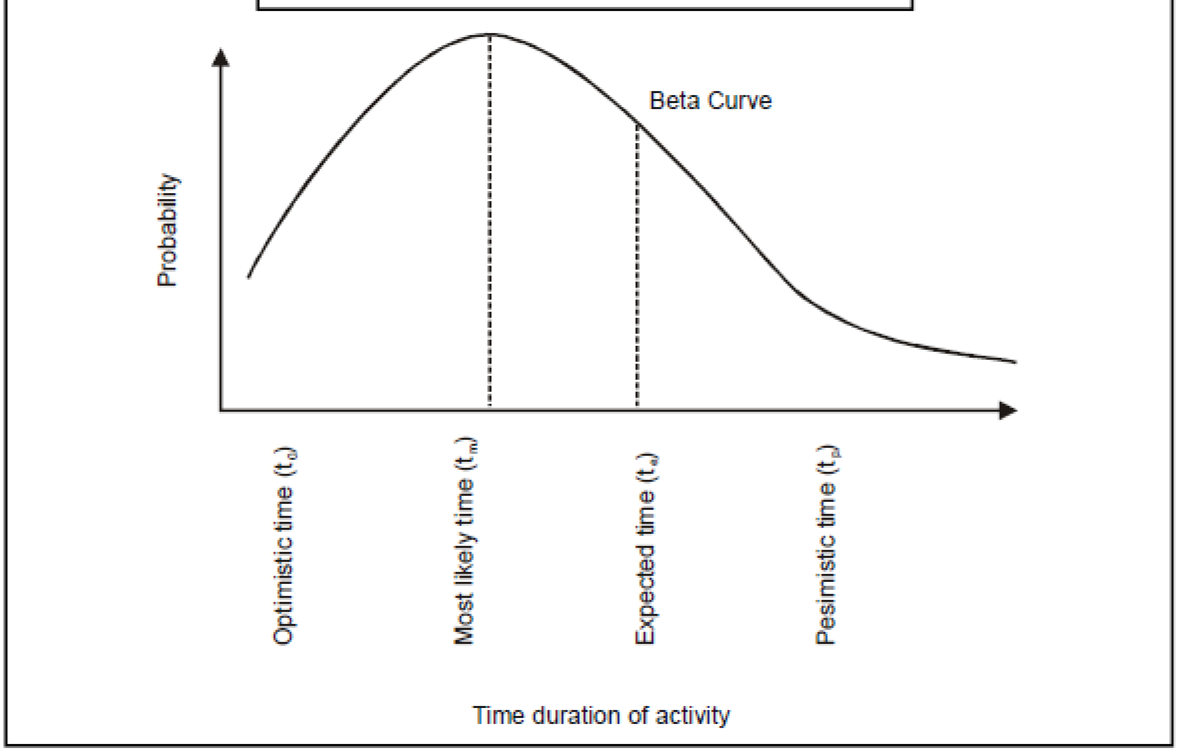
Table 12.7: Calculation of Total and Free Floats

Activity	Normal time (days)	Earliest Time		Latest Time		Float	
		Start	Finish	Start	Finish	Total	Free
1-2	20	0	20	0	20	0	0
1-3	25	0	25	5	30	5	5
2-3	10	20	30	20	30	0	0
2-4	12	20	32	23	35	3	3
3-4	5	30	35	30	35	0	0
4-5	10	35	45	35	45	0	0

12.4 Project Evaluation Review Technique (PERT)

In the critical path method, the time estimates are assumed to be known with certainty. In certain projects like research and development, new product introductions, it is difficult to estimate the time of various activities. Hence PERT is used in such projects with a probabilistic method using three time estimates for an activity, rather than a single estimate, as shown in Figure 12.18.

Figure 12.18: PERT using Probabilistic Method with 3 Time Estimates



Optimistic time t_0: It is the shortest time taken to complete the activity. It means that if everything goes well then there is more chance of completing the activity within this time.

Most likely time t_m: It is the normal time taken to complete an activity, if the activity were frequently repeated under the same conditions.

Pessimistic time t_p: It is the longest time that an activity would take to complete. It is the worst time estimate that an activity would take if unexpected problems are faced.

Taking all these time estimates into consideration, the expected time of an activity is arrived at.

The average or mean (t_a) value of the activity duration is given by,

$$T_a = \frac{t_0 + 4t_m + t_p}{6} \tag{5}$$

The variance of the activity time is calculated using the formula,

$$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2 \tag{6}$$

Probability for Project Duration

The probability of completing the project within the scheduled time (T_s) or contracted time may be obtained by using the standard normal deviate where T_e is the expected time of project completion.

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma^2 \text{ in critical path}}} \tag{7}$$

Probability of completing the project within the scheduled time is,

$$P\,(T \leq T_s) = P\,(Z \leq Z_0) \text{ (from normal tables)} \tag{8}$$

Example: An R&D project has a list of tasks to be performed whose time estimates are given in the Table 12.8, as follows.

Time expected for each activity is calculated using the formula (5):

$$T_a = \frac{t_0 + 4tm + tp}{6}$$

$$= \frac{4 + 4(6) + 8}{6} = \frac{36}{6} = 6 \text{ days for activity A}$$

Similarly, the expected time is calculated for all the activities.

The variance of activity time is calculated using the formula (6).

$$\sigma_i^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

$$= \left(\frac{8 - 4}{6}\right)^2 = 0.444$$

Similarly, variances of all the activities are calculated. Construct a network diagram and calculate the time earliest, T_E and time Latest T_L for all the activities.

Figure 12.19: Network Diagram

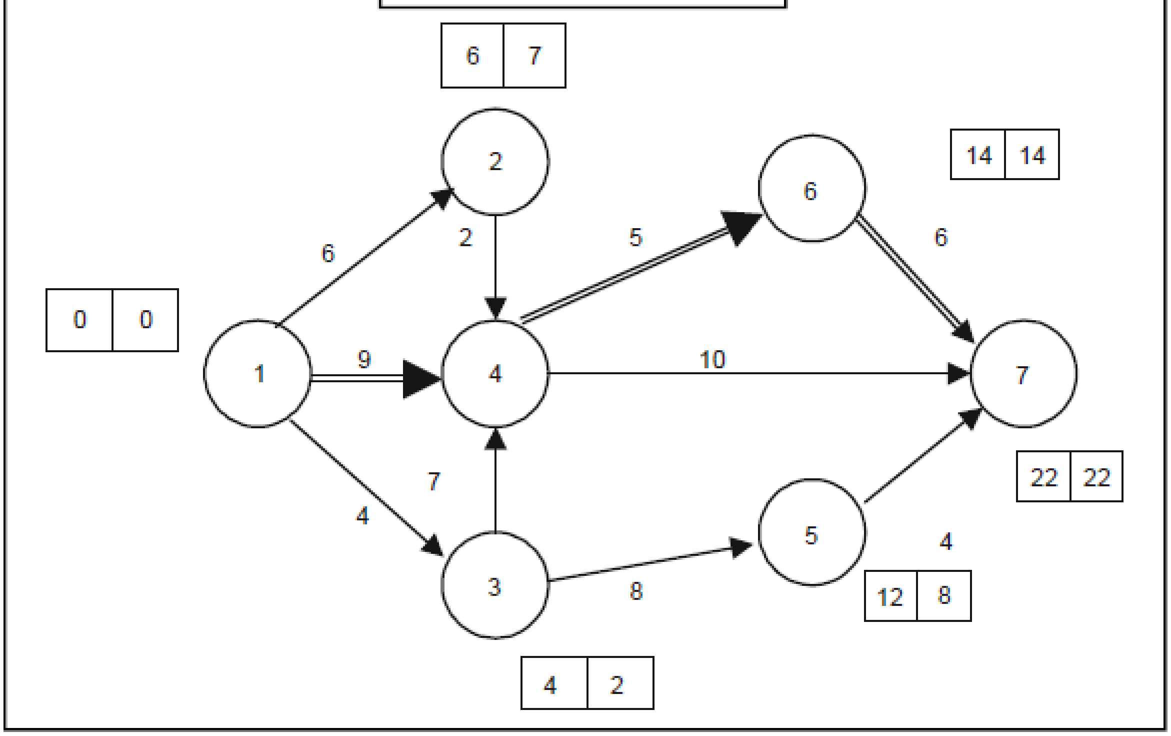


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Table 12.8: Time Estimates for R&D Project

Activity I-j	Activity Name	T_o	t_m (in days)	t_p
1-2	A	4	6	8
1-3	B	2	3	10
1-4	C	6	8	16
2-4	D	1	2	3
3-4	E	6	7	8
3-5	F	6	7	14
4-6	G	3	5	7
4-7	H	4	11	12
5-7	I	2	4	6
6-7	J	2	6	10

1. Draw the project network.
2. Find the critical path.
3. Find the probability that the project is completed in 19 days. If the probability is less that 20%, find the probability of completing it in 24 days.

Solution:

Calculate the time average t_a and variances of each activity as shown in Table 12.9.

Table 12.9: T_e & σ^2 Calculated

Activity	T_o	T_m	T_p	T_a	σ^2
1-2	4	6	8	6	0.444
1-3	2	3	10	4	1.777
1-4	6	8	16	9	2.777
2-4	1	2	3	2	0.111
3-4	6	7	8	7	0.111
3-5	6	7	14	8	1.777
4-6	3	5	7	5	0.444
4-7	4	11	12	10	1.777
5-7	2	4	6	4	0.444
6-7	2	9	10	8	1.777

From the network diagram Figure 12.19, the critical path is identified as 1-4, 4-6, 6-7, with a project duration of 22 days.

The probability of completing the project within 19 days is given by, $P(Z \leq Z_0)$

To find Z_0,

$$Z_0 = \left(\frac{T_s - T_e}{\sqrt{\Sigma\sigma \text{ in critical path}}} \right)$$

$$= \left(\frac{19 - 22}{\sqrt{2.777 + 0.444 + 1.777}} \right)$$

$$= \left(\frac{-3}{\sqrt{5}} \right) = -1.3416 \text{ days}$$

we know, $P(Z < Z_0) = 0.5 - \psi(1.3416)$ (from normal tables, $\Psi(1.3416) = 0.4099$)

$= 0.5 - 0.4099$

$= 0.0901$

$= 9.01\%$

Thus, the probability of completing the R&D project in 19 days is 9.01%. Since the probability of completing the project in 19 days is less than 20%, we find the probability of completing if in 24 days.

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma \text{ in critical path}}}$$

$$= \left(\frac{24 - 22}{\sqrt{5}} \right) = \left(\frac{2}{\sqrt{5}} \right) = 0.8944 \text{ days}$$

$P(Z \leq Z_0) = 0.5 - \Psi(0.8944)$ (from normal tables, $\Psi(0.8944) = 0.3133$)

$= 0.5 + 0.3133$

$= 0.8133$

$= 81.33\%$