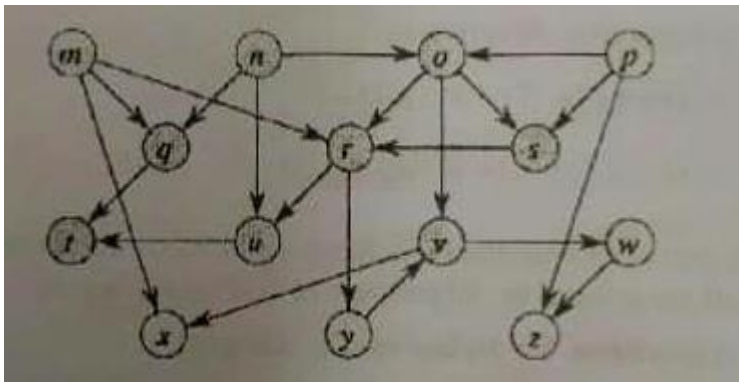
Suggestive Questions on module 1 (Graph) and module 2 (Divide and Conquer)

Question No.	Question	Marks
1	The cost of building a Heap from an array of size n is (i) O(nlogn), (ii) O(logn), (iii) O(n), (iv) O(n^2)	2
2	Give an example of a function f(n) such that f(n) $\in \Omega(n^2)$ .Give an example of a function g(n) $\in \omega(n^2)$.	2
3	Which searching method is most suitable to find a word from a dictionary? What is its worst case time complexity?	2
4	Given n integers, show that [(3n-4)/2] comparisons are sufficient to find their minimum and maximum.	4
5	Apply Master's Method to find T(n) where T(n)=36T(n/6+n√n).	4
6	Write an O(n) algorithm to find the i th largest integer from an array of n integers.	4
7	Write the algorithm for merge sort. Prove that time complexity to sort n elements for merge sort is O(n log n).	3+3
8	Consider k arrays (k >2) each of size n and sorted in ascending order. Give an O(nk logk) algorithm to merge them into a single sorted array.	4
9	Is it possible to sort n elements in less than O(n log n) time in the worst case? Justify your answer.	3
10	Define O(g(n)) and 0(g(n))	2
11	'Finding the least element is asymptotically faster than finding the k th (k>1) element from an unsorted array.' True or False ? Justify.	2

12	What is the minimum number of comparisons required to find the maximum and minimum among n integers?(Your expression should hold for both n even and n odd)	2
13	Write an algorithm to increase a k bit binary counter represented as an array of length k. Show that the cost of n such increment operations in O(n).	5
14	State the Master's Theorem.	5
15	Solve the following recurrence relations: (i) T(n)=7T(n/3) +n2 (ii) T(n)=T(n-1)+1/n Assume T(1)=1	5
16	1. Given recurrence relation $T(n) = \begin{cases} 2T(n/3) - \sqrt{n} & ,n > 1 \\ \theta(1) & ,otherwise \end{cases}$ Find T(n) by applying Master Theorem.	3
17	What do you mean by $y(n) = O(x(n))$, where y, x are functions of n?	2
18	Given $f(n) = an^2 + bn + c$ where $a > 0$, prove that $f(n) = \Theta(n)$ using the definition of $\Theta(n)$.	5
19	Given the recurrence relation $C_n = n + 1 + \frac{2}{n}\sum_{i=0}^{n-1} C_i$ for $n > 0$ and C_0=0 find C_n in terms of H_n.	5
20	Give an algorithm to select the k th smallest element among n elements in time 0(n) in the worst case. Derive a recurrence relation for running time of the algorithm and then prove that it is 0(n).	3+4+3
21	Solve the following recurrences: (i) Let f(n) = 8f(n/2)+n^2, f(1) = 1 (ii) Let f(n) = 4f(n/2) + n^2logn ,f(1) =1	5+5
22	Give the exact formula (as a function of n) for the number of times the word Hello will be printed by Algorithm Helloprint() below:	6

	Algorithm 1 : *HelloPrint(n: integer)* 1 **for** $i \leftarrow 1$ *to* $4n^2 + 1$ **do** 2 **for** $j \leftarrow 1$ *to* i **do** 3 print("Hello"); 4 **end** 5 **end**	
23	Write a divide and conquer version of linear search function divconqlinsearch(int A[],int key) in an array of n integers by dividing the input elements into approximately two halves. What is the time complexity of this algorithm?	4
24	(i) Sort using Radix-sort : 329, 457, 657, 839, 436, 720, 355 . (ii) There are n integers in range [1….k]. Write an algorithm that process the input in 0(n+k) time and then returns. How many numbers there are in range [a…b] in 0(1) time for any given a and b?	4+6
25	(i) You are given two arrays A and B of size n each. Write a 0(logn) time algorithm to find the median of the merged array. (ii) Derive the lower bound of comparison based sorting.	6+4
26	Show the ordering of vertices produced by topological sort when it is run on the DAG below. Assume the DFS procedure considers the vertices in alphabetical order and assume that each adjacency list is ordered alphabetically. 	5
27	a) Let T(n) denote the number of terms in the recursion tree during computation of Fibonacci number using recursive algorithm. Then prove that for $n \geq 2$, $T(n) > 2\text{^}n/2$	5+6+3

	b) Explain the following: Every case time complexity, worst case time complexity, average case time complexity, best case time complexity. C) Prove that n is not is $\Theta(n^2)$	
28	a) If g(n) belongs to sample-0 of $f(n)$, then prove that g(n) belongs to $O(f(n)) - \Omega(f(n))$ b) If f is $O(h)$ and g is $O(k)$, then prove that fg is $O(h^k)$	6+4
29	Let f be an increasing function that satisfy the recurrence relation f(n) = af(n/b) +c Whenever n is divisible by b, where a$\geq$ b, b is an integer greater than 1 and c is a positive real number then prove that $f(n) = \begin{cases} 0(n^{log_b a}) & if a > 1 \\ 0(logn) & if a = 1 \end{cases}$	7
30	Define selection problem.write a parallal algorithm for selection problem.write all associated algorithms, illustrate.	14
31	a) Explain the terms: Every –case time complexity, worst-case time complexity, Average-case time complexity and Best-case time complexity. b) Compute the average case complexity of sequential search algorithm and quicksort algorithm.	6+3+5
32	a) Define the following notations: Big-oh, Big-omega, and Big Theta. b) Prove that if 0<=r<s, then n^t is O(n^3) and n^3 is not O(n^t). c) Prove that if f is O(R) and g is O(R) ,then f+g is O(R). d) Prove that n is in O(n^2) but not in Ω(n^2).	4+3+3+4