2015

COMPUTER SCIENCE – HONOURS

Third Paper

Full Marks – 100

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Answer ***Question No. 1*** and ***any five*** from the rest, taking at least ***two*** from **Group A** and at least ***one*** from each of the other **Groups B** and **C**.

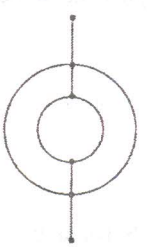
1. Answer ***any ten*** questions : 2×10

(a) When is a graph called a universal graph ? Give a suitable example.

(b) State the Konnisberg bridge problem in Graph Theory.

(c) What is the relation between the total number of vertices and the number of pendant vertices of a tree with at least two vertices ? Justify.

(d) Count the number V of vertices, E of edges and the number R of regions of the map given below and verify Euler's formula. Also find the degree of the outside region.



(e) A farmer buys 3 cows, 2 pigs and 4 hens from a man who has 6 cows, 5 pigs and 8 hens. How many choices does the farmer have ?

(f) What are existential and universal quantifiers ? Illustrate with suitable examples.

(g) What do you mean by complexity of an algorithm ? What are the two cases that are usually investigated in complexity theory ?

(h) Define the rate or order of convergence of an iterative procedure. What is the order of convergence of Newton-Raphson method ?

(i) State the condition for convergence of Gauss-Jacobi method.

(j) Find the relative percentage error in approximate representation of $4/3$ by 1.33.

(k) What are the advantages of using R-K method for numerical solutions of differential equations over Taylor's Series method ?

(l) Design a DFA that accepts strings defined over $\Sigma = \{0,1\}$ whose decimal representation is divisible by 2.

(m) Give the analytical definition of a non-deterministic finite automaton (NDFA).

(n) When is a grammar said to be ambiguous ?

(o) What are the limitations of a finite state automaton ?

Group – A
(Discrete Mathematical Structures)

2. (a) State and prove the necessary and sufficient conditions for a graph G to be an Euler graph. 2+4

(b) Prove that a simple graph with n vertices and k components can have atmost $(n-k)(n-k+1)/2$ edges. 3

(c) Write an algorithm for breadth first search traversal of a graph. How will you find the number of connected components of a given graph by the above algorithm ? 5+2

3. (a) Define a circuit matrix B for a graph G. Illustrate with a proper example. List some of the vital observations that can be made about the circuit matrix $B(G)$. 2+4+2

(b) Consider functions $f(x), g(x)$ and $h(x)$ such that $f(x) = 0(g(x)), g(x) = 0(h(x))$. Show that $f(x) = 0(h(x))$. 4

(c) Show that $(p \to r) \vee (q \to r)$ and $(p \vee q) \to r$ are logically equivalent. 4

4. (a) State and prove the generalised pigeonhole principle. 2+3

(b) Find the number of subsets (with 10 elements) of the set $S=\{1,2,.....,100\}$ that do not contain a pair of consecutive integers. 4

(c) An unbiased coin is tossed five times. Find the probability of getting more than one Head. 4

(d) What do you mean by a proposition and tautology ? Give examples. 3

5. (a) State Cook's theorem. 2

(b) Find the number of primes less than 200 using the principle of Inclusion and Exclusion. 6

(c) Define a minimum spanning tree for a given weighted undirected graph G. Describe the Kruskal's algorithm. Clearly state your assumptions. 2+6

Group – B

(Numerical Methods and Algorithms)

6. (a) Write down the composite expression for Simpson's $\frac{1}{3}^{rd}$ rule. Evaluate $\int_1^2 \frac{dx}{\sqrt{1+x^2}}$, by taking 8 intervals using this rule. Compute the error in this case. 2+6+2

(b) Derive the Newton-Raphson method to find the root of a transcendental equation. State the necessary and sufficient conditions for the convergence of this method. 4+2

7. (a) Given the following table, find $f(x)$ assuming it to be a polynomial of degree three in x. Use Lagrange's Interpolation formula. 6

x	0	1	2	3
y	1	2	11	34

(b) Solve the system of equations by Gauss elimination method correct upto two significant figures. 6

$$x+3y+2z=5$$
$$2x-y+z=-1$$
$$x+2y+3z=2$$

(c) Write an algorithm to find the solution of an equation $f(x)=0$ using Secant method. 4

Group – C

(Formal Languages and Automata Theory)

8. (a) Prove that $L=\{a^p \mid p \text{ is a prime}\}$ is a non-regular using the Pumping Lemma for regular languages. 4

(b) Design a PDA that accepts all palindromes over $\sum=\{a,b\}^*$. 3

(c) Write a brief note on Chomsky classification of grammars. 6

(d) Define a Turing machine. 3

9. (a) Design a DFA that would accept strings with an even number of a's and an odd number of b's over $\sum=\{a,b\}$. 3

(b) Prove that regular languages are closed with intersection and complementation. Hence prove that they are closed under union as well. 7

(c) Give the regular expression for the strings belonging to the following languages. 2+2+2

(i) $\alpha_1=\{x\in\{a,b\}^* \mid x \text{ has no two consecutive } a\text{'s}\}$

(ii) $\alpha_2=\{x\in\{a,b\}^* \mid x \text{ ends with } abb\}$

(iii) $\alpha_3=\{x\in\{a,b\}^* \mid x \text{ has a substring } bab \text{ somewhere}\}$