2015

MATHEMATICS — HONOURS

Third Paper

(Module – V)

Full Marks – 50

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Group – A
(Modern Algebra – II)
(Marks – 15)

Answer ***any three*** questions

1. (a) Prove that every proper subgroup of the symmetric group S_3 is cyclic. 2

(b) Prove that if 'p' be a prime and 'a' be an integer such that 'p' is not a divisor of 'a' then 'p' divides $a^{p-1}-1$. (Use Group theoretic method) 3

2. (a) Define permutation group. Give an example. 1+1

(b) Let $\alpha = (1\ 2\ 5\ 7)$ and $\beta = (2\ 4\ 6) \in S_7$ where S_7 is the set of all permutations on the set $\{1, 2, 3, \ldots, 7\}$. Find $\alpha \circ \beta \circ \alpha^{-1}$; '$\circ$' having its usual meaning. 3

3. (a) Prove that a finite commutative ring R with more than one element and without zero divisors is a field. 3

(b) Find whether the set $S = \{x \in R \mid x \cdot a = 0\}$ is a subring of a ring R where a is a fixed element of R. 2

4. Considering the set of all matrices $M = \left\{ \begin{pmatrix} a & b \\ 2b & a \end{pmatrix}; \text{a, b rational} \right\}$ as a ring under matrix addition and multiplication, show that M forms a field. Also check when a, b $\in$ R, M is a field under the same operation or not. 3+2

5. (a) Show that in the ring $(\mathbb{Z}_n, +, \cdot)$, if an element m is a unit, then the gcd (m, n) is 1. 2

(b) Prove that the ring of Gaussian integers is an Integral domain. 3

Group – B
(Linear Programming and Game Theory)
(Marks – 35)

Answer ***any five*** questions

6. (a) Define a convex cone. 1

(b) Let S be a set of points $\vec{r} = (x, y, z)$ satisfying the equation $3x^2 + 5y^2 - 4z^2 = 0$. Prove that S is a cone but not a convex cone. 1+2

(c) Discuss whether the set A consisting of points on the straight line y = mx + c is convex and find the corresponding convex hull. 1+2

7. An agricultural farm has 180 tons of nitrogen fertilizers, 250 tons of phosphate and 220 tons of potash. The farm will be able to sell 3 : 3 : 4 mixtures of these substances at a profit of Rs. 15 per ton and 2 : 4 : 2 mixtures at a profit of Rs. 12 per ton respectively. Formulate the problem in the form of an L.P.P. to obtain the maximum profit. 7

8. Show that (2, 1, 3) is a feasible solution of the set of following equations :

$$4x_1 + 2x_2 - 3x_3 = 1$$
$$6x_1 + 4x_2 - 5x_3 = 1$$

Reduce it to a B.F.S. 2+5

9. (a) If $\mathbf{x}_0$ be any feasible solution of the primal, max $Z_x = \mathbf{cx}$ subject to $\mathbf{Ax \leq b, x \geq 0}$ and $\mathbf{w}_0$ be any feasible solution to its dual problem, min $Z_w = \mathbf{b'w}$ subject to $\mathbf{A'w \geq c'}$, $\mathbf{w \geq 0}$ then prove that $\mathbf{cx}_0 \leq \mathbf{b'w}$. [Where notations have their usual meaning] 3

(b) Find the dual of the following problem

$$\text{minimise } Z = x_1 + x_2 + x_3$$
$$\text{subject to } 3x_1 - x_2 + x_3 = 4$$
$$2x_1 + x_2 - x_3 \leq 8,$$

x_1 is unrestricted in sign, $x_2 \geq 0$ and $x_3 \geq 0$. 4

10. Using Two-phase method, find the maximum value of $2x + y - z$ subject to

$$4x + 6y + 3z \leq 8$$
$$3x - 6y - 4z \leq 1$$
$$2x + 3y - 5z \geq 4$$
$$x, y, z \geq 0.$$

7

11. Four products are produced in three machines and their profit margins are given by the table below :

	P_1	P_2	P_3	P_4	
m_1	6	4	1	5	14
m_2	8	9	2	7	18
m_3	4	3	6	2	7
	6	10	15	8	39

Capacity

Requirements

Find a suitable production plan of products in machines so that the capacities and requirements are met and profit is maximized.

Do you anticipate any alternative solution ? Give reason. 6+1

12. (a) Write down the Mathematical Formulation of an Assignment problem. 2

(b) Find the optimal assignment cost from the following cost matrix : 5

	1	2	3	4	5
A	–2	–4	–8	–6	–1
B	0	–9	–5	–5	–4
C	–3	–8	–9	–2	–6
D	–4	–3	–1	0	–3
E	–9	–5	–8	–9	–5

13. (a) State the maximin or minimax theorem. 2

(b) Find the optimal strategies and the value of the game for the following game problem : 3

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 3 & 2 & 3 \\ 6 & 2 & 7 \\ 5 & 1 & 4 \end{bmatrix} \end{array}$$

(c) Find the saddle point, if there be any, for the following game problem whose pay-off matrix is given as $A = \begin{pmatrix} -2 & 0 & -1 \\ -5 & 7 & 8 \end{pmatrix}$. Also find the value of the game. 1+1

14. (a) State the Fundamental Theorem of a rectangular game. 2

(b) Solve the following game problem graphically whose pay-off matrix is given as : 5

$$A = \begin{pmatrix} 2 & 2 & 3 & -1 \\ 4 & 3 & 2 & 6 \end{pmatrix}.$$