2015
MATHEMATICS — HONOURS
Fourth Paper
(Module – VIII)
Full Marks – 50

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Group – A
(Marks – 15)

1. Answer ***any one*** from the following :

(a) Find the equation of the circle on the sphere $x^2+y^2+z^2=49$ whose centre is at (2, – 1, 3). 3

(b) Find the cartesian co-ordinates of the point whose spherical polar co-ordinates are $\left(3, \frac{2\pi}{3}, -\frac{\pi}{6}\right)$. 3

2. Answer ***any two*** from the following :

(a) Find the condition of tangency of a plane $lx+my+nz=p$ to a central conicoid $ax^2+by^2+cz^2=1$. Hence find the locus of point of intersection of three mutually perpendicular tangent planes. 3+3

(b) Show that the perpendiculars from the origin on the generators of the paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{2z}{c}$ lie on the cones $\left(\frac{x}{a}\pm\frac{y}{b}\right)(ax\pm by)+2z^2=0$. 6

(c) (i) Show that $4x^2+6y^2-16x+12y-24z+22=0$ represents elliptic paraboloid with vertex (2, – 1, 0). 3

(ii) Prove that the only ruled paraboloid is the hyperbolic paraboloid. 3

(d) A variable sphere passes through $(0, 0, \pm c)$ and cuts the lines $y=x$, $z=c$ and $y=-x$, $z=-c$ in points whose mutual distance is $2a$. Show that the centre of this sphere lies on the circle $z=0,\ x^2+y^2=\left(a^2-c^2\right)$. 6

Group – B

(Marks – 10)

3. Answer ***any one*** from the following :

(a) (i) A particle is resting in equilibrium on a curve at P under the action of forces $\frac{\mu}{r}$ and $\frac{\mu}{r'}$ towards two fixed points O and O' respectively such that $OP = r$, $O'P = r'$. Find the locus of P. 5

(ii) A uniform heavy elliptical wire of semi-axes a, b is hung over a small smooth rough peg. Show that if the wire is in equilibrium with any point on it in contact with the peg, the coefficient of friction must not be less than $\left(\frac{a^2 - b^2}{2ab}\right)$. 5

(b) (i) A system of coplanar forces has total moments H, 2H respectively about $(2a, 0)$ and $(0, a)$. The total resolved parts of the forces along $y = x$ vanishes. Find the points in which the line of action of the resultant meets the co-ordinate axes. 6

(ii) Explain 'angle of friction' and 'cone of friction'. 4

Group – C

(Marks – 25)

4. Answer ***any one*** from the following :

(a) A ball falls from a height h upon a fixed horizontal plane. Show that the whole distance traversed before the ball finished rebounding is $\frac{1+e^2}{1-e^2}.h$ and the time taken is $\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$, '$e$' being the coefficient of restitution. 7

(b) Find the loss of kinetic energy due to direct impact of smooth spheres of masses m_1 and m_2 moving with velocities u_1 and u_2 respectively in same direction. Discuss the cases for perfectly elastic as well as inelastic collision. 7

5. Answer ***any two*** from the following :

(a) (i) A smooth sphere collides with another one of same mass at an angle 45° with the line of impact. If the coefficient of restitution is $^1/_2$, find the angle through which the direction of motion turns. 6

(ii) For a particle starting from rest at a distance 'a' from a fixed point O on a straight line and moving with an acceleration always towards O, varying as distance from O, find the period of oscillation. 3

(b) (i) Define terminal velocity for a particle falling from rest at a height h above earth surface under gravity in a medium whose resistance varies as cube of its velocity. Find the velocity attained by it when it reaches the ground and the time taken by it. 6

(ii) If in a motion of two dimension, radial and cross-radial components of acceleration are equal, find the equation of the path. 3

(c) (i) A particle moves in a straight line from rest under an attractive force μ/y^2 directed towards 'O' on the line, and 'y' is the distance from O. If it starts from a point at a distance $2a$ from O, then prove that it will be at a distance 'a' after a time $\left(\pi/2 + 1\right)\left(\frac{a^3}{\mu}\right)^{1/2}$. 5

(ii) A particle describes the curve $r = a(1 - \cos\theta)$ in constant angular velocity ω. Find the acceleration resolved parallel to initial line and towards the origin. 4

(d) (i) A particle falls under gravity, starting from rest. A resistance equal to k times the square of the velocity acts on it. Show that the kinetic energy acquired is approximately $mgx(1 - kx)$, if k is small. 5

(ii) A particle describes the catenary $y = c\cos h\left(x/c\right)$ under a force which is always parallel to the Y-axis. Find the law of force. 4