2015
PHYSICS – HONOURS
Fourth Paper
(Group – A)
Full Marks – 50

The figures in the margin indicate full marks
Candidates are required to give their answers in their own words as far as practicable

Answer ***Question No. 1*** and ***any four*** from the rest

Symbols have their usual meanings every where

Given $h = 6.626\times10^{-34}$ J-sec.
$m_e = 9.1\times10^{-31}$ kg.
$c = 3\times10^{8}$ m/sec.

1. Answer ***any five*** of the following : 2×5

(a) Find the number of photons emitted per second by a 40 Watt source of light of wavelength 6000Å.

(b) What is the implication of the relation $\left[\hat{H}, \hat{L}\right] = 0$?

(c) Show that if ψ be an eigen function of the operator $\hat{A}$ with eigen value λ, then it is also eigen function of e^{A} with eigen value e^{λ}.

(d) Show that in an isobaric process, the change in enthalpy is equal to the heat transferred between the system and the surrounding.

(e) Show that the specific heat at constant volume is related to the second derivative of the Helmholtz– free energy.

(f) What do you mean by quasistatic process ? All the reversible processes are quasistatic but not all quasistatic processes are reversible — Explain.

2. (a) What is the momentum representation of the position operator $\hat{x}$? Show that this representation satisfies the position-momentum commutation relation.

(b) Examine whether the operator $\hat{B}$ is linear or not, where $\hat{B}\psi(x) = \psi^*(x)$.

(c) Show that the operator

$$\hat{Q} = \frac{1}{2}(\hat{x}\hat{p}_x + \hat{p}_x\hat{x}) \text{ is Hermitian.}$$

(d) Show that if F is Hermitian,

$U = e^{iF}$ is unitary. (2+2)+2+3+1

3. (a) Consider the scattering of photon by an electron initially at rest. Show that the change in wavelength is equal to $\frac{h}{m_e c}(1-\cos\theta)$ where θ is the scattering angle. Discuss the appearance of two peaks in the scattered spectra.

(b) Show that the de Broglie wavelength of an electron is equal to its Compton wavelength when its speed is $c/\sqrt{2}$.

(c) An electron has a de Broglie wavelength of 0.15Å. Compute the phase and group velocities of that wave. (4+1)+2+3

4. (a) State and prove uncertainty principle.

(b) For any operator $\hat{A}$ which has no explicit time dependence, follows

$$\frac{d}{dt}\langle \hat{A} \rangle_t = \frac{i}{\hbar}\left\langle \left[\hat{H}, \hat{A}\right] \right\rangle_t$$

hence prove that $\frac{d}{dt}\langle P_x \rangle = -\left\langle \frac{dV(x)}{dx} \right\rangle$ for a particle moving in x direction with momentum P and under the potential V(x).

(c) An electron of energy 200 eV is passed through a circular hole of radius 10^{-4} cm. What is the uncertainty introduced in the angle of emergence ? (1+3)+3+3

5. (a) Describe the working of a Carnot engine operating between two sources at temperatures T_2 and T_1. What is the need to have two sources at different temperatures ?

(b) Calculate the maximum efficiency of a Carnot engine working between two temperatures $T_2 = 400^oC$ and $T_1 = 120^oC$.

(c) Using the inequality

$$0 \geq \oint dQ/T,$$

show that the entropy of an isolated system undergoing an irreversible process will always increase. (3+2)+2+3

6. (a) If E_s and E_r are the adiabatic and isothermal elasticity constants of a gas and γ is the ratio of two specific heats then prove that $\gamma = E_s/E_r$.

(b) Prove the following relations where symbols have their usual significance :

(i) $\left(\frac{\partial T}{\partial V}\right)_S = \left(\partial P/\partial S\right)_V$

(ii) $\frac{(\partial P/\partial T)_S}{(\partial P/\partial T)_V} = \frac{\gamma}{\gamma - 1}$.

(c) An infinitesimal amount of work can not be represented by an exact differential" — Explain the meaning of this statement. 4+(2+2)+2

7. (a) In what respect does an ideal gas differ from a real gas ? When does this difference become negligible ?

(b) Indicate the co-existence curves and the critical point in a soild-liquid-gas phase diagram.

Show schematically the variation of the order parameter with temperature in a (i) first order (ii) a continuous phase transition.

(c) Starting from the Claussius-Clapeyeron equation for the slope of the vapour pressure curve

$$\frac{dP}{dT} = \frac{L}{T\Delta v}$$

where L is the specific latent heat of evaporation and Δv denotes the specific volume change involved in the phase transition, show that for below the critical point and for vary low pressure, one gets the relation

$$\ln\left(\frac{P_1}{P_2}\right) = \frac{L}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

where (P_1, T_1), (P_2, T_2) are two points on the coexistence curve. 3+(2+2)+3