2016

STATISTICS – HONOURS

Fifth Paper

(Group – B)

Full Marks – 50

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer ***any four*** questions :

(a) What do you mean by a contrast ? When are two different contrasts called orthogonal to each other ? Show that maximum possible number of orthogonal contrasts of $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is $n-1$. 1+1+3

(b) Define a linear model. Discuss when it becomes an ANOVA model, a regression model and an ANCOVA model. 2+3

(c) Give example of a test where

(i) power is unity but size is zero,

(ii) both power and size is ½,

(iii) size is ½ but power is zero. 5

(d) Establish the relation between Mann-Whitney and Wilcoxon rank-sum test statistics. 5

(e) Let X be a random variable with $P\sim [\ X=x] = f(x)$. Consider the problem of testing $H_0 : f = f_0$ against $H_1 : f = f_1$ where

x	1	2	3	4	5
$f_0(x)$	1/5	1/5	1/5	1/5	1/5
$f_1(x)$	1/6	1/4	1/6	1/4	1/6

Find sizes and powers of MP tests for various values of the cut-off points. 5

(f) Suppose linear regression is assumed in each of the two independent paired data sets. Under normal error assumptions, discuss testing procedure to test whether the two lines intersect on Y-axis. 5

(g) If ANCOVA model with a significant regression coefficient holds for a two-way classified data, derive the form of the average variance of all possible estimated elementary contrasts. 5

[Turn Over]

(h) Consider a random sample of size n drawn from a bivariate normal population with correlation coefficient ρ. Provide a likelihood ratio test for testing $H_0 : \rho = 0$ versus $H_1 : \rho \neq 0$. (*Derivation of the maximum likelihood estimates of the parameters are not needed*) 5

Answer ***any two*** questions :

2. (a) Suppose the life-time of a bulb is distributed exponentially with mean life θ (in hours). Let X_i denote the number of trials required to get a bulb surviving at least t (known) hours for the first time in the ith lot, where lot sizes are large enough. Discuss a likelihood ratio test for testing $H_0 : \theta \geq \theta_0$ against $H_1 : \theta < \theta_0$.

(b) Define a uniformly most accurate unbiased (UMAU) confidence set. State and prove the theorem describing the relation between an UMPU test and an UMAU confidence set. 9+6

3. (a) Show that in a two-way $(p \times q)$ classified data, any estimated row contrast is orthogonal to any estimated column contrast.

(b) What do you mean by valid error in the context of ANOVA ? Consider a random effect model for a two-way classified data with m (≥ 2) observations in each cell. Find the expectations of the related mean sum of squares and discuss the role of valid errors for testing the significance of different effects. 5+10

4. (a) Discuss a testing procedure for testing whether the proposal of a linear regression for yield of paddy on rainfall, temperature and amount of manure used is worthwhile or not.

(b) Based on age data for a random sample of individuals in a tribal community, suppose one is interested to test whether the average age of persons in the community is 70 or more than 70. Suppose no specific assumption is made on the probability distribution of age except that the distribution function is absolutely continuous and symmetric.

(i) Discuss a suitable testing procedure.

(ii) Assuming that the null hypothesis is true, find the expectation and variance of the test statistic and check whether the distribution of the test statistic is symmetric or not. 5+(3+(4+3))

5. (a) Consider a random sample from a standard exponential distribution with mean θ.

(i) Provide an UMP size-α test for testing $H_0 : \theta = \theta_0$ (known) against $H_1 : \theta > \theta_0$.

(ii) Find the power function of the test and hence show that the test is unbiased.

(iii) Also get a $100(1-\alpha)$ % lower confidence Bound for θ.

(b) Discuss an application of a randomised test in case of a continuous probability distribution. (4+3+3)+5